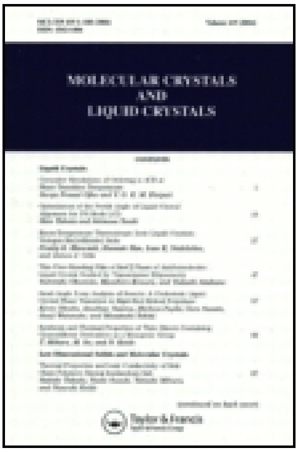

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Quantum Limit for Observation of Self-switching Effect of Light in Nonlinear Spatially Inhomogeneous Optical System

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The quantum theory of nonlinear dynamics and self-switching effect for two coupled waves in nonlinear distributed feedback systems (DFB) – cholesteric liquid crystals and/or spatially inhomogeneous optical fibers (waveguides) – has developed for the first time. The quantum limits of observation of self-switching effect for the polarization characteristics of light have been founded.

Keywords self-switching, DFB-systems; standard quantum limit; nonclassical states of light; quantum gates; optical transistors.

1. INTRODUCTION

Nowadays the problem of fundamental restrictions on optical information processing and transmission by physical systems represents doubtless interest in quantum optics and laser physics [1]. Such a fundamental restrictions are connected with quantum fluctuations of an optical field and determine limiting characteristics of mesoscopic optical devices (transistors, logic gates etc.) of new generation proposed recently for information processing. Indeed, for the present facilities special photon correlation technique gives the possibility to synthesize an entanglement for single photons and to measure the Bell states (see e.g. [2,3]).

In the present paper we analyze quantum properties of light switchers and optical transistors "operating" on macroscopic quantum states of light. We will consider the interaction of distributevely-coupled two modes in Kerr-like nonlinear medium. Physically, such a systems can be realized in optics [4-7], solid state physics [8] and atom optics [9]. Indeed in our previous papers we have shown that both linear and nonlinear energy exchange between coupled waves leads to the possibility of generation of nonclassical (squeezed) light in cholesteric liquid crystals (CLC) [4], tunnel-coupled and/or spatially-periodic fibers/waveguides [5]. The strong correlation between photons can be also achieved due to Raman-Nath scattering in the nematic liquid crystals [6].

The main physical essence of the system under consideration is possibility of "phase transition" in such a system with high nonlinearities and large photon number, taking into account the photon jumps with small number of particles. Thus, an analysis of quantum fluctuations of two-mode optical system under the conditions of strong nonlinear dynamics will be find out.

2. NONLINEAR DYNAMICS OF OPTICAL SYSTEM OF DISTRIBUTEVELY-COUPLED WAVES UNDER THE HARTREE APPROXIMATION

Let's describe the interaction of two distributevely-coupled modes with the same frequencies in Kerr-like nonlinear medium by an interaction Hamiltonian (see e.g. [4,5]):

$$H_I = \frac{\hbar}{2}\left(a_1^+ a_2 + a_2^+ a_1\right) + \frac{\hbar}{2}\left\{\omega_1\left[\left(a_1^+\right)^2 a_1^2 + \left(a_2^+\right)^2 a_2^2\right] + 2\omega_{12} a_1^+ a_1 a_2^+ a_2\right\} \quad (1)$$

where $\omega_1, \omega_{12} \sim \chi^{(3)}$ – dimensionless frequencies describing self-modulation and cross-interaction processes correspondingly. The annihilation (creation) operators a_j (a_j^+) are satisfy to usual commutation relations:

$$\left[a_i; a_j^+\right] = \delta_{ij}, \qquad i, j = 1,2 \,. \tag{2}$$

The first term in expression (1) characterizes linear energy exchange (photon tunneling) between two modes, and second one describe their nonlinear interaction.

Let us consider the nonlinear dynamics of the system in Schrödinger representation for two practically important cases:

(i) interaction of entangled polarization modes with a fixed total number of photons N :

$$|\Psi(\tau)\rangle_N = \frac{1}{\sqrt{N!}}\left(\alpha_1(\tau)a_1^+ + \alpha_2(\tau)a_2^+\right)^N |\mathbf{0}\rangle, \tag{3a}$$

(ii) two-mode interaction in coherent field approximation (two-mode coherent states):

$$|\Psi(\tau)\rangle_{coh} = \mathrm{e}^{-\frac{N_0}{2}} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{\left[\sqrt{N_0}\alpha_1(\tau)\right]^{n_1}}{\sqrt{n_1!}} \frac{\left[\sqrt{N_0}\alpha_2(\tau)\right]^{n_2}}{\sqrt{n_2!}} |n_1\rangle|n_2\rangle, \tag{3b}$$

where $|\mathbf{0}\rangle \equiv |0\rangle_1|0\rangle_2$. The functions $\alpha_1(\tau)$ and $\alpha_2(\tau)$ can be find out using standard variational method. They are obey to normalization condition $|\alpha_1(\tau)|^2 + |\alpha_2(\tau)|^2 = 1$

Finally we arrive to the equations in terms of real variables ζ and ϕ:

$$\frac{d\zeta}{d\tau} = -\sqrt{1-\zeta^2}\sin\phi\,, \tag{4a}$$

$$\frac{d\phi}{d\tau} = \frac{\zeta}{\sqrt{1-\zeta^2}}\cos\phi + B\zeta\,, \tag{4b}$$

where $\zeta \equiv \dfrac{\langle a_1^+ a_1\rangle - \langle a_2^+ a_2\rangle}{N} = |\alpha_1|^2 - |\alpha_2|^2$ is relative photon number (population imbalance) in polarization modes; $\phi \equiv \theta_2 - \theta_1$ is phase difference. In (4) B is the switching parameter, defined by expression:

$$B = (N-1)(\omega_{12} - \omega_1). \quad (5)$$

The system (4) has three sets of stationary solutions:

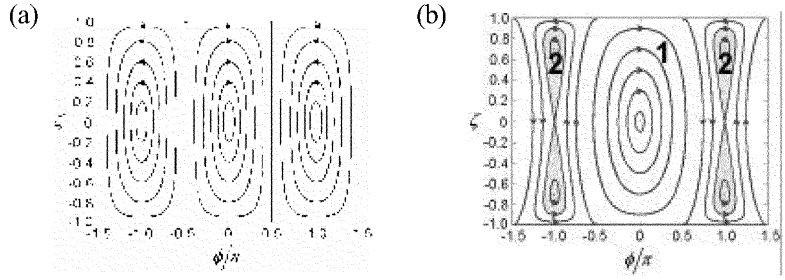
$$\zeta^{(0)} = 0, \qquad \phi^{(0)} = \pi k, \qquad k = 0, \pm 1, \pm 2, \ldots \quad (6)$$

and for $|B| > 1$:

$$\zeta^{(\pm)} = \pm\sqrt{1 - \frac{1}{B^2}},$$
$$\phi^{(\pm)} = \pi(2k+1), \text{ for } B > 0 \quad (7)$$
$$\phi^{(\pm)} = 2\pi k, \qquad \text{for } B < 0, \ k = 0, \pm 1, \pm 2, \ldots.$$

The stationary point (7) is a center, and the type of the solution (4) is determined by the value of parameter B. For the case $|B| < 1$, the singular point (6) also still center, and for $|B| > 1$ it becomes saddle (unstable equilibrium point).

Let's briefly consider the dynamical regimes for the system (4) on phase plane (ζ, ϕ). There exist three different patterns of phase portraits. When the condition $|B| < 1$ is satisfied (Figure 1a) the phase trajectories are closed around the stationary points $(0; \pi k)$, $k = 0, \pm 1, \pm 2, \ldots$. Physically, that corresponds to periodic variation of average number of particles in each of interacting modes, for which the value ζ varies in the limits: $-1 \le \zeta \le 1$ ($\mathrm{sgn}[\zeta(\tau)] \in \{-1, 0, 1\}$). If the parameter B is changing in the region of $1 < |B| < 2$ (Figure 1b), there exist new areas on phase plane relevant to different regimes of energy exchange between modes. In the area 1 the behavior of system is similar to the case in Figure 1a. In the area 2 variances of average photon number also is periodic, however, there is no "full energy exchange" between two modes ($\mathrm{sgn}[\zeta(\tau)] = 1$ or $\mathrm{sgn}[\zeta(\tau)] = -1$). Such



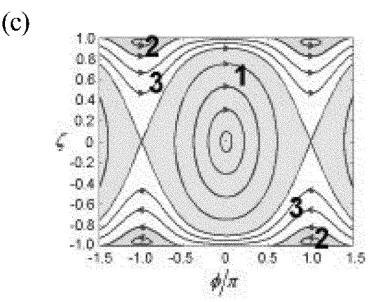


FIGURE 1. Phase portraits for canonically conjugated parameters ζ and ϕ for different values of B: a) $B = 0.1$ ($|B| < 1$), b) $B = 1.4$ ($1 < |B| < 2$), c) $B = 3.5$ ($|B| > 2$).

interaction regime corresponds to self-trapping of photons in the cores of fiber. The phase trajectories are closed around stationary points $\left(\pm\sqrt{1-\frac{1}{B^2}};2\pi k\right)$, $k = 0,\pm1,\pm2,\ldots$.

In the third case, for which $|B| > 2$ (Figure 1c), there is one more set of solutions describing dynamics of average photon numbers without full energy exchange between modes, as well as solution in the area 2. However the phase difference ϕ now is not periodically vary (phase trajectories are not closed) in analogy with mechanical pendulum rotatio n.

3. THE EFFECT OF SELF-SWITCHING OF QUANTUM POLARIZATION STATES OF LIGHT

Let us consider the effect of self-switching of light polarization. Although the self-switching of light intensity for two-mode systems is well known in classical optics [7], the physical interpretation of this effect should be more quantum, than classical. Actually, it is necessary to consider the photon tunneling process in a fiber as macroscopic quantum effect like Josephson effect for superconductors, for which Cooper pairs are tunnel between two junctions [5].

So, we define the self-switching (switching) of a quantum state of light as a phenomenon, for which the small variation of an initial photon state (3) (determined by values ζ_0, ϕ_0, N) or medium properties (parameters ξ, ω_{12}), accompanying with abrupt transition between phase trajectories which are taking place on the different sides of separatrix (see Figure 1). In this case the interaction time (length of medium) is still fixed.

We will describe the polarization properties of two-mode optical system by the Stokes parameters [5]:

$$S_0 = a_1^+ a_1 + a_2^+ a_2, \qquad S_1 = a_1^+ a_1 - a_2^+ a_2 \qquad \text{(8a,b)}$$

$$S_2 = a_1^+ a_2 + a_2^+ a_1, \qquad S_3 = \mathrm{i}\left(a_2^+ a_1 - a_1^+ a_2\right). \qquad \text{(8c,d)}$$

The operators S_j obey to SU(2) algebra commutation relations:

$$[S_2, S_3] = 2\,\mathrm{i}\,S_1, \quad [S_1, S_2] = 2\,\mathrm{i}\,S_3, \quad [S_3, S_1] = 2\,\mathrm{i}\,S_2 \qquad \text{(9a)}$$

$$[S_j, S_0] = 0, \qquad j=1,2,3. \qquad \text{(9b)}$$

Thus, simultaneous and precise measurement of noncommuting Stokes parameters is impossible due to presence of quantum fluctuations in optical system.

In the Figure 2 the effect of self-switching is presented for average values of the Stokes parameters S_j and variance of fluctuations $\zeta^{(0)}=0$ as the function of B parameter for fixed interaction time $\tau = 2\pi$. In this case $\delta B \sim \delta N$ (see (5)); $\delta N = \left|N^{(2)} - N^{(1)}\right|$ is input photon number for the self-switching effect; the numbers of photons $N^{(2)}$ and $N^{(1)}$ correspond to values of parameters $B^{(2)}$ and $B^{(1)}$. It is easy to see that the self-switching area of the parameter S_1 is

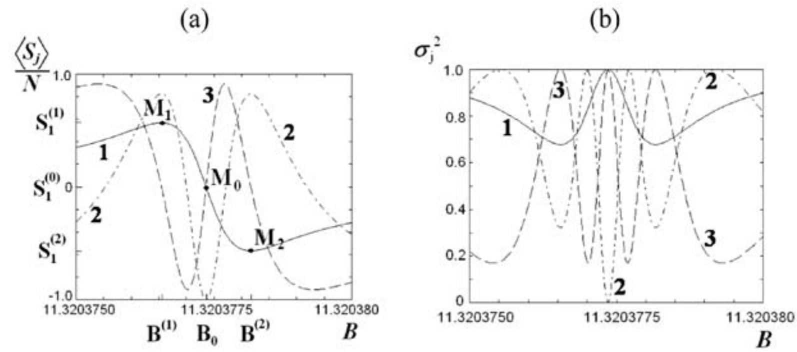


FIGURE 2. Dependences for (a) normalized average values of the Stokes parameters $\langle S_j \rangle / N$ and (b) normalized variance of fluctuations of the Stokes parameters $\sigma_j^2 \equiv \langle \Delta S_j^2 \rangle / N$ ($j = 1,2,3$) for the state (3a) against the parameter B. Initial values are: $\zeta_0 = -0.1$, $\phi_0 \approx 1.103\pi$, $\tau = 2\pi$. Numbers on the figure correspond to numeration of the Stokes parameters.

determined by points M_1 and M_2, and the critical value of control parameter $B_c \equiv B_0 \approx 11.32037735$.

Obviously, considered above nonlinear dynamics and self-switching effect can be observable only in the case when the fluctuations of input and output photon number are not essential.It is easy to see in Figure 2b that output variances of the fluctuations of Stokes parameters. $\langle \Delta S_j^2 \rangle$ are suppressed below the coherent level of fluctuations $\langle \Delta S_j^2 \rangle = N$ ($j = 1,2,3$) that corresponds to formation of essentially nonclassical polarization-squeezed light.

For the input quantum state of optical system following relation should be fulfilled:

$$\delta N >> \sqrt{\langle \Delta N^2 \rangle} \qquad (10)$$

Using the expressions (5), (10) it is possible to introduce the effective L parameter of the problem, which is determine the condition for the self-switching effect observation that takes into account the quantum fluctuations of light at the input of medium:

$$L \equiv \frac{\sigma_N}{\sqrt{N_0}} \frac{B_0}{\delta B} << 1, \tag{11}$$

where B_0 corresponds to the middle switching point in Fig.2; $\delta B = \left| B^{(2)} - B^{(1)} \right|$ is the "width" of switching area (see Figure 2a); N_0 is the average particle number corresponded to the value B_0, $N_0 \approx B_0 / (\omega_{12} - \omega_1)$ at $N >> 1$; σ_N is normalized variance of fluctuations of input photon number N_0: $\sigma_N = \sqrt{\langle \Delta N_0^2 \rangle / N_0}$.

For a coherent state ($\sigma_N = 1$), quantity $L \equiv L_{SQL} = B_0 / \delta B \sqrt{N_0}$ determines standard quantum limit of self-switching effect observation.

We deal with classical effect of self-switching when $L_{SQL} << 1$. In the case, for which relation $L_{SQL} \approx 1$ is take place , the observation of self-switching is principally impossible within the framework of the classical approach. In this case an inequality (11) can be fulfilled using nonclassical states at the input of medium.

Here we give simple numerical estimations for such a condition. Firstly, let's consider tunnelly-coupled silica optical fibers with refractive index $n = 1.46$, linear coupling coefficient $K = 3.6 \cdot 10^{-7}$, nonlinear coefficient $\Theta \sim 10^{-12}$ CGSE , core cross-section $S \sim 10^{-7}\,\mathrm{cm}^2$, wavelength of radiation $\lambda = 0.53 \mu\mathrm{m}$. We have $n = 1.8$, $K = 3 \cdot 10^{-1}$, $\Theta \sim 10^{-4}$ CGSE, $S \sim 10^{-7}\,\mathrm{cm}^2$, $\lambda = 1.06 \mu\mathrm{m}$ for cholesteric liquid crystal (CLC) [4]. Here we also suppose that $\Theta_{12} - \Theta_1 = \Theta$. For self-switching of the Stokes parameter S_1 from Figure 2 we have $B_0 / \delta B \approx 7.13 \cdot 10^6$ that corresponds to the value of pump photon number $N_0 \approx 1.58 \cdot 10^{13}$ for silica fiber, and $N_0 \approx 10^7$ for a CLC accordingly; the interaction lengths are $z_{\mathrm{sil}} \approx 2.15\,\mathrm{m}$,

$z_{\mathrm{CLC}} \approx 0.006\ \mathrm{mm}$. So, the standard quantum limit for observation of self-switching effect is $L_{SQL,\mathrm{sil}} \approx 1.79$ for silica fiber, and $L_{SQL,\mathrm{CLC}} \approx 2 \cdot 10^3$ for the CLC. Thus, here squeezed state of light (with total photon number N) is absolutely necessary for the switching effect observation in CLC.

4. CONCLUSION

Thus, in our work we developed the quantum theory of two-mode interaction of distributively coupled waves in spatially inhomogeneous optical systems.

The obtained results has a great practical interest for creating of optical equipment of new generation for quantum communication and information transmission. Namely we note here optical transistors and logic gates operating on essentially quantum signals (nonclassical states of light) with small number of photons for the switching. The most important result here is determination of quantum fundamental limits on observation and measurement of self-switching effect.

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